ISSN 1066-369X, Russian Mathematics, 2018, Vol. 62, No. 10, pp. 26–29. © Allerton Press, Inc., 2018.
Original Russian Text © B.A. Kats, S.R. Mironova, A.Yu. Pogodina, 2018, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2018, No. 10, pp. 29–33.

Singular Integral Equations on Non-Smooth Curves in Hölder–Frechét Space

B. A. Kats[1*], S. R. Mironova[2], and A. Yu. Pogodina[2***]**

[1]*Kazan Federal University*
ul. Kremlyovskaya 18, Kazan, 420008 Russia

[2]*Kazan National Research Technical University*
ul. K. Marksa 10, Kazan, 420111 Russia

Received August 22, 2017

Abstract—We consider singular integral equations on non-smooth rectifiable curves and on certain classes of curves of infinite length.

DOI: 10.3103/S1066369X18100031

Keywords: *singular integral equation, non-smooth curve, Hölder condition, Frechét space.*

We usually call the equations

$$a(t)f(t) + \frac{b(t)}{\pi i}\ \text{v. p.} \int_\Gamma \frac{f(\tau)d\tau}{\tau - t} + Kf(t) = c(t), \quad t \in \Gamma,$$

where Γ is a curve given on complex plane, a, b, and c are functions given on this curve, f is a function on Γ that we want to find, and K is a given compact operator the singular integral equations. The integral operator which we understand in the sense of its principal value

$$\mathcal{S}f(t) := \frac{1}{\pi i}\ \text{v. p.} \int_\Gamma \frac{f(\tau)d\tau}{\tau - t}$$

is called the singular integral with Cauchy kernel [1, 2]. The classic results on this topic [1, 2] were obtained under assumption that the contour Γ is smooth and functions f, a, b, and c satisfy the Hölder condition

$$h_\nu(f;\Gamma) := \sup\left\{\frac{|f(t_1) - f(t_2)|}{|t_1 - t_2|^\nu} : t_{1,2} \in \Gamma, t_1 \neq t_2\right\} < \infty \tag{1}$$

with exponent $\nu \in (0, 1]$. Under these assumptions the singular integral $\mathcal{S}f(t)$ exists in any point $t \in \Gamma$, and the image $\mathcal{S}f$ satisfies the Hölder condition with the same exponent when $\nu < 1$ [1–3]. However both of these properties are not valid on non-smooth curves. In this connection we will introduce the following notation.

Let Γ be a simple closed curve on complex plane dividing it onto domains D^+ and D^-, $\infty \in D^-$. Let us study the Cauchy type integral

$$\mathcal{C}f(z) := \frac{1}{2\pi i}\int_\Gamma \frac{f(t)dt}{t - z}, \ \ z \notin \Gamma.$$

It presents the analytic in $\overline{\mathbb{C}} \setminus \Gamma$ function which vanishes in point at infinity. If this function has limit values on Γ from the left and from the right, then we will introduce the notation

$$\mathcal{C}^\pm f(t) := \lim_{D^\pm \ni z \to t \in \Gamma} \mathcal{C}f(z).$$

*E-mail: katsboris877@gmail.com.
**E-mail: srmironova@yandex.ru.
***E-mail: apogodina@yandex.ru.